Tech. Edu. J. 19 (1) : 195-214, Winter 2025

Technology of Education Journal
(TEJ)

Homepage: jte.sru.ac.ir

TEJ

SRTTU

ORIGINAL RESEARCH PAPER

# Enhancing the Teaching of Longitudinal Relationship Theorems in Iranian High School Geometry: Integrating GeoGebra Software and Practical Examples

***M. Mehri Tekmeh[1], M.A. Fariborzi Araghi[*,1], E. Reyhani[2]***

[1] *Department of Mathematics and Computer Science, Central Tehran branch, Islamic Azad University, Tehran, Iran*
[2] *Department of Mathematics, Faculty of Sciences, Shahid Rajaei Teacher Training University, Tehran, Iran*

Received: 11 June 2024
Reviewed: 26 July 2024
Revised: 05 September 2024
Accepted: 30 November 2024

**KEYWORDS:**
Pedagogical Approaches
GeoGebra Software
Longitudinal Relations
Geometry
Example

* Corresponding author
*m_fariborzi@iauctb.ac.ir*
(+98912) 2191207

**ABSTRACT**

**Background and Objectives:** This comprehensive study delves into the comparative effectiveness of diverse educational approaches in enhancing students' comprehension and learning skills, specifically in the domain of geometric theorem proofs. The research focuses particularly on the section related to longitudinal relationships within triangles, a crucial component of the 11th-grade geometry curriculum. To identify the most impactful teaching strategies, this investigation meticulously compares three distinct pedagogical methods: (1) using illustrated examples formulated by the researcher through GeoGebra software, (2) exclusive use of examples, and (3) adherence to traditional teaching methods.

**Materials and Methods:** This research employs a quasi-experimental design with two experimental groups and one control group, including pre-test and post-test assessments. In terms of its objective, it is considered an applied study. The study population comprised 332 female 11th-grade students from District 3 of Isfahan during the second half of the 2021-2022 academic year. For the selection of the statistical sample, 60 students from three schools in District 3 of Isfahan were chosen using a purposive sampling method. All participants completed Cattell's Culture Fair Intelligence Test and a pre-test in Geometry 1 with identical questions. The first experimental group received an instructional video on constructing simple shapes and measuring shape components using GeoGebra software, along with virtual instructions on effective software use. The first author addressed their issues in five one-hour sessions. Additionally, five practical examples related to longitudinal relationships in triangles from the 11th-grade geometry textbook were presented, with responses designed using GeoGebra. Content validity was ensured through feedback from five experienced professors and teachers, who also reviewed aspects such as adherence to Persian grammar. The test questions demonstrated good reliability (Cronbach's alpha = 0.81). Given that this number exceeded 0.7, the test questions demonstrate good reliability. After confirmation and correction, the examples were provided to both experimental groups consecutively over five weeks, with 90-minute sessions conducted in the classroom. Examples were given to students without answers. The first group used GeoGebra to solve examples, hypothesize relationships between components, and verify them. Subsequently, correct answers were provided, and students were required to infer and prove the principles of relevant theorems with teacher guidance. Data analysis was conducted using SPSS 25 software, including ANCOVA analysis, Kolmogorov-Smirnov test, and Levene's test.

**Findings:** ANCOVA (Analysis of Covariance) results revealed statistically significant differences among the groups. Notably, the primary experimental group, which utilized a combination of carefully selected examples and interactive GeoGebra software, demonstrated superior performance compared to the second experimental group. This enhanced performance was particularly evident in two critical areas of geometric understanding: the accurate identification of relevant geometric theorems and the subsequent application of these theorems to complex problem-solving scenarios.

**Conclusions:** This comprehensive research underscores the significant benefits of integrating examples and GeoGebra software in geometry education. The findings demonstrate that this combined approach not only enhances students' understanding but also fosters their creativity and promotes active participation in the learning process. By engaging with interactive, visual representations of geometric concepts, students are empowered to explore, hypothesize, and verify mathematical ideas independently. This increased engagement ultimately leads to a more robust and lasting comprehension of geometric principles. It is recommended that educators actively encourage students to install GeoGebra software on their personal devices and allocate specific time for students to gain proficiency in its use.



| NUMBER OF REFERENCES | NUMBER OF FIGURES | NUMBER OF TABLES |
|---|---|---|
| 61 | 9 | 5 |

مقاله پژوهشی

# تقویت آموزش قضایای روابط طولی در درس هندسه دبیرستان‌های ایران: تلفیق نرم‌افزار جئوجبرا و مثال‌های کاربردی

**منیره مهری تکمه[1]، محمدعلی فریبرزی عراقی*[1]، ابراهیم ریحانی[2]**

[1] گروه ریاضی و علوم کامپیوتر، واحد تهران مرکزی، دانشگاه آزاد اسلامی، تهران، ایران
[2] گروه ریاضی، دانشکده علوم، دانشگاه تربیت دبیر شهید رجائی، تهران، ایران

تاریخ دریافت: ۲۲ خرداد ۱۴۰۳
تاریخ داوری: ۰۵ مرداد ۱۴۰۳
تاریخ اصلاح: ۱۵ شهریور ۱۴۰۳
تاریخ پذیرش: ۱۰ آذر ۱۴۰۳

**واژگان کلیدی:**
رویکردهای آموزشی
نرم‌افزار جئوجبرا
روابط طولی
هندسه
مثال

*نویسنده مسئول
m_fariborzi@iauctb.ac.ir
۰۹۱۲-۲۱۹۱۲۰۷

## چکیده

**پیشینه و اهداف:** این مقاله به بررسی اثربخشی رویکردهای آموزشی متمایز در افزایش درک و مهارت یادگیری دانش‌آموزان در اثبات قضایای هندسی، با تمرکز بر بخش مربوط به روابط طولی درون مثلث‌ها که بخش مهمی از برنامه درسی هندسه کلاس یازدهم است، می‌پردازد. این تحقیق به طور دقیق سه روش آموزشی متمایز را با هم مقایسه می‌کند: (۱) استفاده از مثال‌های مصور فرموله شده توسط محقق از طریق نرم‌افزار جئوجبرا، (۲) استفاده انحصاری از مثال‌ها، و (۳) پایبندی به روش‌های تدریس سنتی.

**روش‌ها:** این پژوهش یک طرح شبه‌آزمایشی با دو گروه آزمایش و یک گروه کنترل است که شامل پیش‌آزمون، پس‌آزمون و از نظر هدف کاربردی می‌باشد. دانش‌آموزان پایه یازدهم ریاضی را به سه گروه مجزا شامل دو گروه آزمایش و یک گروه کنترل تقسیم کردیم. جامعه آماری این پژوهش، کلیه دانش‌آموزان دختر پایه یازدهم دوره دوم متوسطه شهر اصفهان در ناحیه سه در نیمه دوم سال تحصیلی ۲۰۲۲-۲۰۲۱ به تعداد ۳۳۲ دانش‌آموز بود. برای انتخاب نمونه آماری ۶۰ دانش‌آموز از سه مدرسه ناحیه سه اصفهان از روش نمونه‌گیری هدفمند استفاده شد. همه گروه‌ها تحت آزمون هوش کتل سه مقیاسی با گروه آزمایشی اول قرار گرفتند. پیش‌آزمون هندسه ۱ شامل سؤالات یکسان برای همه گروه‌ها اجرا شد. گروه آزمایشی اول، فیلم آموزشی ساخت اشکال ساده و اندازه‌گیری اجزای شکل با استفاده از نرم‌افزار جئوجبرا به همراه دستورالعمل مجازی نحوه استفاده مؤثر از نرم‌افزار را دریافت کردند. نویسنده اول مقاله اشکالات آنها را در پنج جلسه یک‌ساعته رفع کرد. علاوه بر این، پنج مثال کاربردی مربوط به کتاب هندسه پایه یازدهم با تمرکز بر روابط طولی در مثلث‌ها ارائه شد. پاسخ آنها با استفاده از نرم‌افزار جئوجبرا طراحی شد. برای بررسی اعتبار محتوا در فرایند ساخت، از نظرات پنج استاد و معلم مجرب استفاده شد. مواردی مانند رعایت دستور زبان فارسی با اعمال نظرات و دیدگاه‌های پنج استاد و معلم مجرب بازنگری شد. برای تعیین پایایی سؤالات از ضریب آلفای کرونباخ استفاده شد و پایایی آن ۰/۸۱ بود. با توجه به اینکه این عدد بیش از ۰/۷ شد، سؤالات آزمون از پایایی خوبی برخوردار هستند. پس از تأیید و تصحیح، مثال‌ها به طور متوالی در طی پنج هفته در اختیار هر دو گروه آزمایشی قرار گرفت و جلسات ۹۰ دقیقه‌ای در کلاس درس انجام شد. مثال‌ها به دانش‌آموزان بدون پاسخ داده شد. گروه اول برای حل مثال‌ها، فرضیه‌سازی روابط بین اجزا و بررسی صحت آنها از نرم‌افزار جئوجبرا استفاده کردند. متعاقباً پاسخ‌های صحیح در اختیار ایشان قرار گرفت و موظف شدند با راهنمایی معلم اصل قضایای مربوطه را استنباط کرده و اثبات کنند. برای تجزیه و

تحلیل داده‌های به دست‌آمده از بخش استنباطی نرم‌افزار SPSS 25 که شامل تحلیل آنکوا، آزمون کولموگروف اسمیرنوف، آزمون لون بود استفاده شد.

**یافته‌ها:** نتایج ANCOVA (تحلیل کوواریانس) تفاوت آماری معنی داری را بین گروه ها نشان داد. شایان ذکر است، گروه آزمایش اول که از تلفیق مثالهای انتخاب شده دقیق و نرم‌افزار تعاملی جئوجبرا استفاده کردند، نسبت به گروه آزمایشی دوم عملکرد بهتری از خود نشان دادند. این عملکرد بهبود یافته به ویژه در دو حوزه مهم درک هندسی مشهود بود: شناسایی دقیق قضایای هندسی مرتبط و کاربرد بعدی این قضایا در روشهای حل مسئله پیچیده.

**نتیجه‌گیری:** این تحقیق جامع بر مزایای قابل توجه ادغام مثال ها و نرم افزار جئوجبرا در آموزش هندسه تاکید دارد. یافته‌ها نشان می‌دهد که این رویکرد ترکیبی نه تنها درک دانش‌آموزان را افزایش می‌دهد، بلکه خلاقیت آنها را نیز تقویت می‌کند و مشارکت فعال در فرایند یادگیری را ارتقا می‌دهد. با درگیر شدن با نمایش‌های تعاملی و بصری مفاهیم هندسی، دانش‌آموزان تشویق می‌شوند که به طور مستقل ایده‌های ریاضی را کشف، فرضیه‌سازی و تأیید کنند. این درگیری بهبود یافته در نهایت منجر به درک قوی تر و پایدارتر از اصول هندسی می شود. توصیه می‌شود که مربیان فعالانه دانش‌آموزان را تشویق کنند تا نرم‌افزار جئوجبرا را بر روی دستگاه‌های شخصی خود نصب کنند و زمان خاصی را به دانش‌آموزان اختصاص دهند. تا در استفاده از آن مهارت کسب کنند.


## Introduction

Mathematics is a core scientific discipline with diverse branches, each with distinct characteristics. Algebra focuses on symbol manipulation and analytical problem-solving. Geometry emphasizes the mental visualization of shapes and spatial relationships. Statistics involves collecting, analyzing, and summarizing data [1]. Students' attitudes toward geometry significantly impact their progress and achievements. A positive outlook on geometry leads to greater dedication and commitment to excel [2]. Teachers play a vital role in education, responsible for developing scientifically literate individuals for the 21st century [3]. This influence is especially strong among geometry educators, reaching 80%.

Teachers support students facing learning challenges through scaffolding. As described by Wood [4], scaffolding involves an expert, like a teacher, assisting someone with less knowledge or expertise, such as a student. Scaffolding methods include cue cards, handouts, prompts, hints, examples, questions, narratives, explanations, visuals, and inquiry-based and problem-solving strategies [1]. Educators widely recognize problem-solving as essential to the mathematics curriculum. It should be taught alongside arithmetic skills, which are important for both academic and real-world success [5]. Developing problem-solving skills improves logical, critical, and creative thinking. It's considered a key life skill, involving analysis, interpretation, reasoning, prediction, evaluation, and reflection [6]. Traditional geometry instruction often relies on lectures with one-way communication, focusing on reading, writing, and discussion. However, integrating Information and Communication Technology (ICT) in education requires proficiency beyond these basic skills. In teaching and learning, ICT includes various multimedia technologies such as text, sound, graphics, animation, and simulators [7]. Most research on digital tools in mathematics education focuses on classroom settings, curricula, and school contexts [8]. Mathematics typically uses four main representation modes: (1) verbal, (2) graphical, (3) algebraic, and (4) numeric. While certain representations may be emphasized, transferring between modes remains important [9]. Digital math environments allow students to identify connections, properties, patterns, and rules by manipulating various representations [10]. Teachers' use of digital technology in classrooms depends on factors like personal beliefs, attitudes toward technology, and views on mathematical knowledge and learning [11].

The main goal is to create an environment where students discover mathematical structures through diverse software-aided representations, improving their understanding of concepts that are often difficult to grasp with traditional methods [12]. Geometry is particularly suited for trial-and-error activities [13]. Toluk notes that students are more likely to develop advanced skills when they're interested in the learning activities. Dynamic geometry software, like GeoGebra, can effectively build this interest [13]. These environments offer increased embodiment for technology-enhanced learning, with dragging actions providing motoric engagement and gestural congruency on touchscreen devices [14]. While some educators believe effective math teaching involves patience, repetition, exploration, explanation, and extensive practice [15], dynamic geometry software significantly contributes to learning. Teachers use their knowledge to choose appropriate technological applications, helping students tackle geometry problems. Selecting a suitable framework is key, allowing teachers to incorporate dynamic applications into active learning environments through careful planning [12].

Education that truly prepares students involves giving them chances to learn independently [16]. Constructivism sees students as active participants who build their own understanding. In this view, teachers guide and support, focusing on student-centered methods [17]. Constructivism puts the individual first in social learning, stressing learning over traditional teaching. It sees knowledge as personal interpretations from interacting with the world, with each person forming their own view of reality. Learning is active, based on experiences, and shaped by what someone already knows. The level of teamwork between teachers and students directly affects how much and how well students learn. Knowledge grows through individual interaction, interpretation, and self-reflection [18]. Both teachers and students take part in activities that fit with constructivist ideas. Windschitl outlined these roles for teachers:

- "Ask about students' beliefs and experiences on key concepts, then create ways for them to actively rethink what they know.
- Give students many chances to work on complex, meaningful problems.
- Help students talk about their work and collaborate with classmates.
- Provide various information sources and tools, including technology and visual aids, to support learning.
- Show students how they think through problems, and encourage students to share their thoughts through talking, writing, art, or other means.
- Often ask students to use what they've learned in new situations, explain their ideas, interpret texts, predict outcomes, and discuss using evidence.
- Use different ways to assess and give feedback on students' beliefs, thinking processes, and results [19]."

Teaching draws on various professional areas, including subject expertise, teaching methods, curriculum knowledge, and information literacy. It also requires cultural awareness and strong communication skills [20]. Education researchers support engaging students in tasks that involve creating and interpreting field-specific representations. They also recommend exposing students to new representations beyond those in typical textbooks [21]. Educational simulations can improve teaching by offering concrete depictions of abstract ideas, aiding understanding. These simulations let students interact with material at their own speed, using their existing knowledge to reshape their mental models [22]. Some simulations are exemplars, which help communicate, explain, and discuss ideas, especially in mathematics. Teachers use exemplars to help students generalize and build mathematical understanding [23]. Learning through examples isn't new; it's been studied in education for decades. From the 1950s to the 1970s, researchers used the learning-by-example approach to study concept formation [24]. Exemplars are crucial for student learning. Expanding knowledge depends on having various examples, knowing how to create them, strengthening their connections, and accessing them easily [23]. While the basis for a conjecture is sometimes clear, it's often not obvious. In these cases, mathematicians turn to examples to explore new problems [25].

Many factors influence teaching geometric drawing and reasoning. Teachers must balance these elements to maximize class effectiveness, using all available professional tools and knowledge [20]. Well-designed examples clearly communicate between learners, teachers, and abstract concepts. In math education, examples serve many purposes: understanding concepts, motivating, proving, reasoning, developing hypotheses, assessing, making connections, generalizing, fostering creativity, and problem-solving. Examples are central to teaching math concepts, advancing math education, and supporting teaching theories [26]. Two key factors in students' problem-solving skills are teaching methods and textbook choice [27]. Curriculum planners and textbook authors should create problems that are both realistic and solvable in multiple ways [28]. Using dynamic graphics in multimedia learning allows students to explore geometric concepts [27]. Multimedia bridges abstract ideas and concrete observations, significantly impacting learning when designed with educational principles and cognitive theories in mind [29]. Computers and digital devices have long helped create visual representations and engage students. Advancing technology allows for integrating multiple representations through various modes, creating more complex visuals [21]. Among available software, GeoGebra is particularly useful for incorporating technology in math education [30]. "GeoGebra can improve problem-solving skills, especially in making logical and creative guesses, by:

- Activating relevant schemas;
- Encouraging the use of problem-solving methods;
- Positively influencing solution process control and individual beliefs [27].

GeoGebra's standout feature is its ability to animate objects and variables, making it an interactive tool for exploring various mathematical outcomes through manipulation. This dynamic software plays a key role in teaching mathematical concepts. GeoGebra, along with programs like Maple and Algebrator, is more than just a math tool; it's designed specifically to improve learning [31]. Different versions of GeoGebra are available for download at www.geogebra.org, compatible with various computer and mobile operating systems.

As a dynamic application, GeoGebra enables students to build and modify geometric and algebraic representations. Its features are designed to improve problem-solving and logical reasoning skills. Students can create mathematical objects based on their questions and hypotheses, deepening their understanding of mathematical concepts [32]. GeoGebra supports multiple languages and offers corresponding online courses, providing comprehensive resources for teachers and students. A dedicated website has been set up to support GeoGebra's practical use [12].

GeoGebra allows teachers to create problems that students can solve using the software, sparking interest and improving understanding of geometric challenges. It encourages active engagement with problem statements and proofs. The software's dynamic nature motivates students to explore mathematics, promoting active participation. As a result, students are more likely to retain knowledge and develop critical thinking and investigative skills. Using GeoGebra's dynamic system helps develop professional competencies for future math educators, especially in information and communication. Examples show that GeoGebra promotes practice-oriented math education and research methods and increases student motivation. The ability to create and study interactive models in GeoGebra's learning environment improves the efficiency of science and math learning, fostering logical thinking and increasing student motivation [33]. Integrating technology in math education allows students to develop diverse and innovative solutions. This approach lets students form and test hypotheses while building mathematical knowledge. Teachers can use GeoGebra to create examples that encourage students to seek answers and spark interest in the subject. This method strengthens students' understanding of geometric problems, enabling them to analyze problem statements and provide logical proofs independently. Previous research examined teaching the chapter on circles and geometric transformations in the Iranian high school Geometry 2 textbook [33]. This study focuses on teaching a different section of the same textbook, specifically longitudinal relationships in triangles (pages 61 to 76). We aim to assess the effectiveness of two teaching methods: using GeoGebra software with practical illustrations, and using researcher-designed examples without software. The study addresses these research questions:

○ To what degree does the integration of GeoGebra software and the inclusion of illustrative examples bolster the comprehension of geometric theorems within the chapter covering longitudinal relationships in triangles found in the Geometry 2 textbook?

○ To what extent does the application of researcher-designed examples, independently created without the assistance of software, facilitate the grasp of geometric theorems in the chapter concerning longitudinal relationships in triangles within the Geometry 2 textbook?

## Review of the Related Literature

The learning of mathematics is complex, requiring an understanding of various mathematical concepts [34]. It involves engaging with real-world phenomena through hands-on experiences [34]. Real Mathematics Education became the main approach to mathematics education in the Netherlands starting in 1968 [35]. Between 1987 and 1997, textbooks aligned with Real Mathematics Education grew from 15% to 75% of the market [35]. Recently, the Ministry of Education in Iran formed a commission of teachers to review and update the math curriculum to better prepare students for their personal and professional futures [35]. Research shows that interest and a positive attitude are key to success in mathematics. Oldknow and Taylor [36] note that motivating students through verbal and non-verbal methods is the first step in effective teaching. Students should understand what they're learning and why. One way to boost motivation is by connecting learning materials to students' everyday experiences [37]. A study of 1083 Chinese math teachers found that they often use technology to engage students during lesson planning [38]. Another study of 40 male middle school students in Tehran, Iran, showed that computer games create an engaging environment that improves problem-solving skills, increasing interest in math and leading to better academic performance [39]. A study of 60 high school students in Badrud, Iran, found that using educational simulations significantly improved students' academic goals and overall learning [40]. Geometry often faces challenges related to student attitudes and achievement. Students tend to show less interest and slower progress in geometry compared to other math topics. Geometry is prone to misconceptions and learning difficulties, requiring a deeper understanding for effective learning [41]. Students recognize the importance of teaching methods in their learning process and are aware that different teachers use different approaches. About 60% of students attribute their learning outcomes to their teachers' teaching style [42]. Many studies in educational technology show that students who use technology in learning achieve higher accuracy and proficiency than those who don't [12]. Research by Guven and Kosa [43] highlighted the positive effects of dynamic geometry software on spatial skills. Similarly, GeoGebra, a dynamic math program, has been shown to develop algorithmic thinking skills by visually representing formal and algorithmic aspects of math problems [44]. This technology allows for the creation of complex geometric structures on computer screens [33]. GeoGebra, an open-source dynamic math software program, was developed as part of Markus Hohenwarter's master's thesis at the University of Salzburg, Austria [45]. The goal was to create a program that combined geometry, algebra, and calculus in one user-friendly package, unlike other software that treated these as separate. After its online release in 2002, GeoGebra gained popularity among educators who used it in their classrooms, winning several educational software awards, including the European Academic Software Award in 2002.

Hohenwarter continued improving GeoGebra, inspired by his doctoral research on the software's educational uses in Austrian schools. In 2006, he joined Florida Atlantic University (FAU) as a visiting professor, participating in the National Science Foundation (NSF)-funded Mathematics and Science Partnership (MSP) teacher education initiative [45]. A study in Ankara, Turkey, found that GeoGebra made learning about lines and angles more enjoyable for 7th-grade students [13]. Another study with 24 mathematics education students at Langlangbuana University in Indonesia showed that those using GeoGebra to learn geometric transformations had significantly better math comprehension than those taught traditionally [46]. Recognizing the importance of the mathematics curriculum, future math educators need strong problem-solving skills. A study of 71 second-semester math education students at Tidar University in Indonesia found that learning spatial geometry with GeoGebra improved their problem-solving abilities [47]. Research with student teachers also showed that GeoGebra effectively improved problem-solving skills in analytical geometry courses. Students responded positively to GeoGebra in these courses, showing high interest in multimedia components but less interest in visual imagery [48]. Using concrete, relevant examples is considered effective for learning math [49]. Historically, mathematicians have used examples to form conjectures by creating systematic lists and looking for patterns [50]. A study of 75 middle school math teachers in Markazi Province, Iran, during the 2010-2011 academic year found that while teachers recognized the importance of examples, they lacked knowledge about educational examples and their teaching potential. Their concept of examples did not include theorems, problems, definitions, proofs, or arguments [51].

Trafton and Reiser [52] compared teaching solely through examples with teaching through examples and problem-solving. They found that presenting both examples and problems had a greater impact on learning than examples alone. Students learn best when they can apply knowledge from examples to new problems. The study concluded that the most effective way to structure learning materials is to present a solved example followed by a similar problem for students to solve, allowing them to recognize similarities and support their problem-solving process. In mathematics, introducing real-world problems connects students to the subject and highlights its everyday relevance. This approach creates purposeful experiences that build motivation and curiosity, ultimately improving understanding of mathematical concepts [53]. Research aimed at improving geometry instruction and creating an immersive learning environment suggests that using various techniques, such as incorporating examples and GeoGebra software, can be a significant step forward.

## Method

### Participants

The study population comprised 332 female 11th-grade students from District 3 of Isfahan during the second half of the 2021-2022 academic year. For the selection of the statistical sample, 60 students from three schools in District 3 of Isfahan were chosen using a purposive sampling method.

The participants were divided into two experimental groups and one control group.

### Instruments

All participants completed Cattell's Culture Fair Intelligence Test and a pre-test in Geometry 1 with identical questions. The first experimental group received an instructional video on constructing simple shapes and measuring shape components using GeoGebra software, along with virtual instructions on effective software use. The first author addressed their issues in five one-hour sessions. Additionally, five practical examples related to longitudinal relationships in triangles from the 11th-grade geometry textbook were presented, with responses designed using GeoGebra. Content validity was ensured through feedback from five experienced professors and teachers, who also reviewed aspects such as adherence to Persian grammar. The test questions demonstrated good reliability (Cronbach's alpha = 0.81). Given that this number exceeded 0.7, the test questions demonstrate good reliability.

### Design

This research employs a quasi-experimental design with two experimental groups and one control group, including pre-test and post-test assessments. In terms of its objective, it is considered an applied study. Two methods were investigated for teaching longitudinal relationships in Geometry 2: (1) solved examples using GeoGebra software and (2) examples created by the researcher without software. This study tested these hypotheses:

Both methods are more effective for theorem learning than traditional approaches. When teaching geometry theorems, particularly Theorem 2 on longitudinal relationships in triangles, solved examples with GeoGebra have a greater impact than instructional examples alone. Data analysis was conducted using SPSS 25 software, including ANCOVA analysis, Kolmogorov-Smirnov test, and Levene's test.

### Procedure

Eleventh-grade math students were divided into three groups: two experimental and one control. The first experimental group used teacher-provided examples and GeoGebra for theorem conjecture and proof. The second group used only teacher-provided examples, while the control group received traditional instruction. The research hypothesis stated: Using examples and GeoGebra software to learn geometric theorems about longitudinal relationships in triangles has a positive effect and is more effective than teaching with examples alone. Figure 4 summarizes the study sessions.

| Teaching topics | Sessions |
|---|---|
| **Teaching topic:** Determining the length of a triangle's side using one side length and two angles | **Problem (1) Description:** Students were presented with a scenario involving a triangular area in Isfahan city, formed by Khayyam, Motahari, and Saeb streets. Given specific angle measurements and one side length, students were tasked with calculating the length of Khayyam Street. |

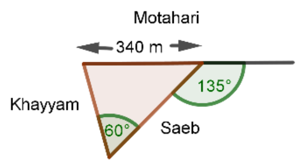



**Fig. 1: Figure drawn in GeoGebra for problem 1**

| Teaching topics | Sessions |
|---|---|
| **Teaching topic:** Determining the length of the third side of a triangle using the lengths of two sides and the included angle | **Problem (2) Description:** Students solved a practical problem involving the installation of a slide in a children's playground. They were required to calculate the distance between the base of a ladder and the base of the slide, given specific angle and length constraints. |
| **Teaching topic:** Calculating the median length of a triangle using the lengths of its three sides | **Problem (3) Description:** This session involved a real-world application where students calculated the total distance of a route between school, library, sports club, and home, given specific distances between locations. |

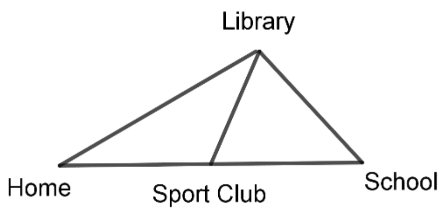



**Fig. 2: Figure drawn in Geogebra for problem 3**

| Teaching topics | Sessions |
|---|---|
| **Teaching topic:** Determining the length of the interior angle bisector in a triangle | **Problem (4) Description:** Students engaged with an astronomy-themed problem, calculating the distance between a celestial object and the Sun, using given distances between Earth, Neptune, and the Sun, and the object's position relative to these bodies. |

**Teaching topic:** Calculating the area of a triangle using the lengths of its sides and applying Heron's formula

**Problem (5) Description:** The final problem involved calculating the total area of a proposed rectangular swimming pool and an adjacent lawn, demonstrating the practical application of area calculations in a real-world context.

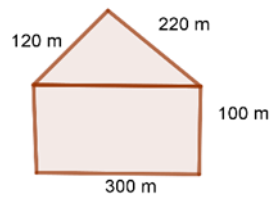



**Fig. 3: Figure drawn in Geogebra environment for problem 5**

**Methodology Note**

All problems were presented to students for solving, with figures drawn in the GeoGebra environment to aid visualization and understanding.

**Fig. 4: Training sessions overview: Geometry 2 Theorems - Longitudinal triangle relationships.**

The questions were based on real-world scenarios. For question 1, Figure 5 shows an aerial image of Motahari, Saeb, and Khayyam streets captured using GPS technology.


**Fig. 5: Motahari Street distance (340 meters) - Real image**

After receiving the questions, students began solving the problems using tools such as protractors and rulers. They also applied their knowledge of trigonometric relationships, which led to accurate solutions in some cases. For example, in problem 1, students recognized that the sine function could determine the length of the side opposite a given angle in a right triangle. Figure 6 shows a correct solution to question one, as presented by one of the students.

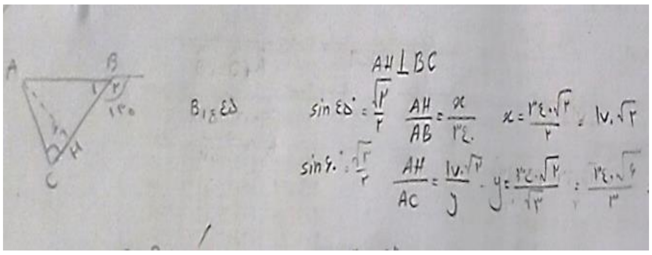


**Fig. 6: Problem (1) solution by the experimental group student**

In the first experimental group, we enhanced educational examples with GeoGebra software. This allowed for visual representations and measurements, simplifying problem-solving. The dynamic GeoGebra environment lets students test various approaches, shifting between experimental inferences and mathematical reasoning. Students could make initial guesses using the software, observe results, and gather information by exploring different scenarios. This process often led to correct answers through empirical reasoning. While mathematical and experimental approaches seemed equally valuable at first, group discussions and teacher prompts helped students recognize the need for formal proofs to support their ideas. After the groups formulated answers, the teacher provided indirect guidance without intervening in their choices. Solutions were validated through software and group consensus before presentation to the teacher, who then discussed them with students as a peer. If the solution withstood scrutiny, it was presented to the entire class. Group members then defended their assumptions and solutions against critiques from the teacher and other groups, highlighting the importance of formal proof. Initially, some students were hesitant to engage in problem-solving. However, successfully resolving problems led to increased excitement and pride. This enthusiasm grew, encouraging even reluctant students to collaborate actively after the second session. GeoGebra's features, which allowed for quick hypothesis testing while working in groups, further encouraged exploration and research. The process of defending claims and critiquing others' solutions not only increased excitement and problem-solving skills but also improved students' accuracy in presentations and responses. Figure 7 illustrates the solution for problem 1.

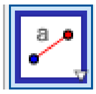
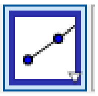
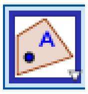
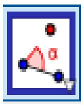
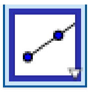
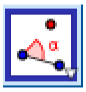
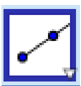
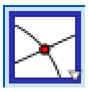
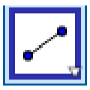
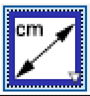
| Tool selection | Procedure |
|---|---|
|  | 1. Construct a line segment AB with a length of 3.4 units. |
|  | 2. From point A, extend a ray passing through line segment AB. |
|  | 3. Select an arbitrary point C on the ray extending from A. |
|  | 4. At point B, construct an angle of 135° in the clockwise direction from the initial line segment AB. |
|  | 5. Extend a ray from point B along the 135° angle constructed in step 4. |
|  | 6. At point A, construct an angle of 75° in the clockwise direction from the initial line segment AB. |
|  | 7. Extend a ray from point A along the 75° angle constructed in step 6. |
|  | 8. Identify the intersection point of the two rays constructed in steps 5 and 7. Label this point D. |
|  | 9. Connect points A and D to form line segment AD. |
|  | 10. Measure and record the length of line segment AD. |

**Fig. 7: Problem (1) step-by-step solution using GeoGebra**

The final output corresponds to Figure 8.

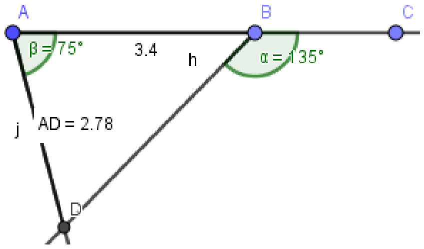



**Fig. 8: Problem (1) final solution output in GeoGebra**

Using a 1:100 scale applied to the figure, we estimated Khayyam Street's length to be approximately 270 meters. This measurement aligns with the length obtained from the aerial map, as shown in Figure 9.


**Fig. 9: Khayyam Street length - Real map measurement**

The second experimental group tackled problems using a pen, paper, ruler, and protractor to sketch shapes. Teacher guidance helped them form hypotheses about angle and side relationships, which they then verified. This hands-on approach led to a deeper understanding of theorem proof.

During this time, the control group learned the same content through standard teaching methods. After the course, all groups took a post-test with 20 questions on length relationships in Geometry Triangle 2. To ensure content validity, experienced Geometry 2 teachers evaluated how well the questions measured the content and objectives.

## Results and Findings

To test our hypothesis that "Using examples and GeoGebra software for teaching geometric theorems related to longitudinal relationships in triangles has a more positive impact than traditional example-based teaching," we used a one-way analysis of covariance (ANCOVA).

This method allowed us to evaluate the effect of the independent variable on the dependent variable while accounting for potential covariate influences. Before applying ANCOVA, we needed to meet several key conditions:

- Random sampling
- Normal distribution of the dependent variable within each group
- Equal variances of the dependent variable across groups
-. Consistent regression line slope between the covariate and dependent variable at different levels of the independent variable

To assess the initial impact of intelligence on geometry learning, we administered the Kettle Scale 3 intelligence test to both experimental and control groups.

Table 1 shows that the mean intelligence scores of students in both experimental and control groups are similar. We then checked whether students' scores in learning geometry concepts were normally distributed.

The data in Table 2 support the assumption of normal distribution for geometry concept learning scores at the 0.05 significance level. We used Lune's test on these scores to assess the impact of the pre-test on post-test results and to check for equal variances.

Table 3 shows that the F-value for geometry concept learning is not statistically significant ($F = 2.773$, $p > 0.05$). This indicates that the group variances are likely homogeneous. For covariance analysis, we must check if the regression line slope between the covariate and dependent variable is consistent across different levels of the independent variable in both groups. This step is necessary to ensure the covariates are not linked to the independent variables.

Table 4 shows that the independent variable's significance level is 0.234 (p > 0.05), indicating it does not significantly affect the dependent variable. The F-values for the interaction between the covariates—pre-test and intelligence (0.080 and 0.560, respectively)—are both above 0.05. This suggests these two variables have little impact on the dependent variable (post-test scores). Therefore, we can conclude that these variables did not significantly influence the post-test scores. The marked improvement in post-test scores appears to be primarily due to the use of examples and GeoGebra software in the experimental groups.

Table 5 reveals several key differences. The average difference between the first experimental group, which used examples and GeoGebra software, and the second experimental group, which used only examples, is 0.975. This difference is significant (p = 0.022, < 0.05), showing that the first group performed better than the second. The first experimental group also outperformed the control group, with an average difference of 1.444 (p = 0.000). In contrast, the average difference between the second experimental group and the control group is 0.469. This difference is not statistically significant (p = 0.557, > 0.05). This suggests that teaching Geometry 2's chapter on longitudinal relationships in triangles using examples alone does not differ significantly from traditional methods.

These findings support the research hypothesis: combining GeoGebra software with examples effectively improves geometry concept understanding among 11th-grade mathematics students.

**Table 1: Intelligence scores comparison - Experimental vs. control groups (mean and standard deviation)**

| | Experiment Group 1 | | Experiment Group 2 | | Control Group | |
|---|---|---|---|---|---|---|
| | Mean | Std. deviation | Mean | Std. deviation | Mean | Std. deviation |
| Intelligence | 106.65 | 11.77 | 107.05 | 15.00 | 106.73 | 12.66 |

**Table 2: Data normality check - Kolmogorov-Smirnov test**

| Components | Group | Statistic | D f | Sig. |
|---|---|---|---|---|
| Learn the Concepts of Geometry | The experiment 1 | .129 | 20 | .200 |
| | The experiment 2 | .123 | 20 | .200 |
| | Control | .075 | 20 | .200 |

**Table 3: Leven-test to check for equality of variances**

| Components | F | Df1 | Df2 | Sig. |
|---|---|---|---|---|
| Learn the Concepts of Geometry | 2.773 | 2 | 57 | .071 |

**Table 4: Between-subjects effects tests**

| D f | Source | Mean Square | F | Sig. |
|---|---|---|---|---|
| 2 | Group | 1.898 | 1.495 | .234 |
| 1 | Pretest | 265.538 | 209.083 | .000 |
| 1 | Intelligence | 1.240 | .976 | .328 |
| 2 | Group*Pretest | .102 | .080 | .923 |
| 2 | Group* Intelligence | .711 | .560 | .575 |

| D f | Source | Mean Square | F | Sig. |
|---|---|---|---|---|
| 51 | Error | 1.270 | | |
| 60 | Total | | | |

**Table 5: Pairwise Comparisons**

| Group(I) | Group(J) | Mean Difference (I-J) | Sig. |
|---|---|---|---|
| Experiment Group 1 | Experiment Group 2 | .975 | .022 |
| | Control Group | 1.444 | .000 |
| Experiment Group 2 | Experiment Group 1 | -.975 | .022 |
| | Control Group | .469 | .557 |

## Discussion

A recent study examined three teaching approaches for the distance relationships chapter in 11th-grade geometry:

- Combining practical, real-world examples designed by the first author with GeoGebra software
- Using these examples independently
- The standard teaching method

Van den Heuvel-Panhuizen and van Zanten [35] argue that students' lack of mathematical understanding often leads to avoidance. They advocate for reality-based mathematics education, which aims to rebuild mathematical concepts using informal knowledge and relatable facts under teacher guidance.Gholam Azad [28] describes this reality-oriented approach as "guided re-creation," where students experience the process of mathematical invention, developing a sense of ownership over their strategies and results. To make geometry more engaging, the first author, an Isfahan resident, drew inspiration from her surroundings. She designed questions to teach geometry theorems related to distance relationships in triangles more effectively. Experienced geometry teachers reviewed these questions. For example, she conceived a problem to estimate the length of Khayyam Street using geometry after crossing the Marnan Bridge near the intersection of Khayyam and Saeb streets. This required knowledge of Motahari Street's length, the distance between Saeb and Khayam streets, and the angles between them. She obtained this information from the "Neshan" route finder program's aerial map. In the geometry class, Question 1 sparked excitement. Some students suggested measuring Khayyam Street in person, viewing it as both an outing and a problem-solving exercise. One student proposed using cycling distance-tracking software. However, the challenge was to solve the problem without leaving the classroom. Geometry provided the solution. The process began with modeling the real-world scenario. Streets were represented as triangle sides, and a graph was drawn using the given angles. Students recalled right-triangle theorems to tackle the problem. In groups using only examples, students worked with pen and paper. But was there a quicker, easier method? Recent years have seen increased student engagement through software use [54]. Research by Mukamba and Makamure [55] shows that integrating information and communication technology, particularly GeoGebra, improves students' understanding of geometric transformations compared to traditional methods. Computer-generated forms and dynamic visuals have improved mathematical concept visualization and representation, outpacing traditional methods in both speed and knowledge retention [54]. Various software applications exist for mathematical problem-solving, including ChatGPT, PhotoMath, Squirrel, Minerva, GeoGebra, and Wolfram Alpha [56].

For geometry problems, GeoGebra stood out as an effective tool for personalizing learning, increasing accessibility, and improving educational assessments [54]. Students, already familiar with GeoGebra from previous lessons, worked in groups of three to model the problem. Using the software's distance calculation feature, they determined the required length. Stronger students assisted those who needed help. The dynamic nature of GeoGebra allowed students to observe how changing angles affected triangle side lengths, encouraging them to hypothesize about side-angle relationships. To conclude, the teacher introduced the sine theorem for triangles. This approach aligns with research by Arbian and Shakur, which confirms GeoGebra's positive impact on student progress [57]. Student participation in problem-solving fostered deeper learning through increased interaction. Shadan and Leong [58] noted that collaborative learning helps students reinforce their knowledge with peer support. Peer interactions enable students to guide each other towards shared understanding. Higher-ability students play a crucial role in helping their lower-ability peers succeed in assignments. This study revealed that GeoGebra software boosts students' motivation to learn geometry. This aligns with Zangin and Tatar's [59] findings that interactive learning environments correlate with cooperative learning in improving student motivation and engagement.

Question 2 was inspired by a real-world scenario: planning a kindergarten next to a school for teachers' children. The challenge was to determine the area needed for a safe, low-height slide. This problem, presented in the second teaching session, introduced the cosine theorem. Notably, some students who previously disliked geometry now looked forward to these classes. However, some viewed the software as a game, requiring careful teacher supervision to maintain educational focus. A student named Maryam, who frequented Horisa Sports Club and planned to visit the Imamzadeh Mohsen (AS) library, inspired Question 3. This problem required calculating a detour route, introducing the concept of midpoint calculations. GeoGebra proved invaluable, offering both speed and accuracy in problem-solving.

Another student, Setayesh, expressed interest in astronomy's geometric applications. This led to Question 4, which incorporated real data about Earth, Neptune, and Sun distances from Google. This problem served to teach the triangle bisector theorem, demonstrating geometry's relevance in astronomical contexts.

The Bakhtiar sports complex, with its swimming pool and green space, inspired Question 5. As the geometry class progressed, student engagement and interest grew, making the subject enjoyable. This outcome aligns with Joshi and Singh's research [60], which found that students view GeoGebra as an effective tool for learning mathematics, grasping concepts, boosting problem-solving confidence, and fostering creativity in a visual, enjoyable manner. However, it's important to note that using GeoGebra effectively requires teachers to have a clear understanding of the shapes they intend to draw. A study of 23 math teachers at a small U.S. university revealed that while they could easily draw familiar shapes, they struggled with less common ones like non-isosceles trapezoids [61]. In the experimental group taught using only designed examples without software, students showed enthusiasm in participation. The use of real-world examples relevant to students' lives increased their motivation to attend and learn geometry. This approach builds on Ozgur's research [24], which highlighted the use of examples for elementary students in exploring, developing, refining, generalizing, and justifying conjectures. The effectiveness of examples in this study aligns with Rezaei and Mohammadzadeh's findings [46]. They identify examples as efficient tools for expressing key features of instructional definitions or explanations in problem-solving. Examples, with their unique characteristics, significantly impact learners' understanding by representing concepts and demonstrating concept formation processes. This research confirmed the hypothesis that combining examples with GeoGebra software for teaching geometric theorems about length relationships in triangles is more effective than using examples alone. This integrated approach encouraged active student engagement, speculation, and a deeper understanding of the subject matter.

## Conclusions

This study examined the effectiveness of different methods for teaching distance relationships in triangles. It compared the use of practical examples solved with GeoGebra software against using practical examples alone and traditional teaching methods. For 11th-grade Geometry 2 students, GeoGebra software facilitated more efficient learning of triangle distance relationships without causing frustration. It improved students' software skills while making the geometry classroom more engaging and enjoyable. Introducing practical examples before using GeoGebra for problem-solving enhanced this effect. This approach improved problem-solving skills and increased student motivation for active participation and collaboration. The findings suggest that geometry education should extend beyond theory to include enriching learning tools. Combining GeoGebra software with practical examples in teaching triangle distance relationships empowers students to explore mathematics, develop critical thinking, and unlock their creative potential. Geometry educators are encouraged to learn GeoGebra and effectively incorporate visual aids into their lessons. Recommendations include:

- Encouraging students to install GeoGebra on their personal devices.
- Allocating time for students to become proficient with the software.
- Math teachers designing applied examples for various geometry topics during their planning meetings.
- Having students solve these examples using GeoGebra to experience a more interactive and practical learning approach. This integrated method promotes a more engaging, practical, and effective geometry learning experience.

## Authors' Contribution

The first author compiled and analyzed data and wrote the article. The second author (Corresponding author) developed the research concept and design. They also guided the article's compilation and review. The third author provided guidance and advice on drafting and revising the article.

## Acknowledgments

The authors thank the anonymous reviewers for their constructive feedback. They also express gratitude to the geometry teachers of Isfahan: Mehri Eetesami, Azar Hossein Reza, Badralsadat Arefi Far, Maryam Mirtalebi, and Ayaz Bahrami Esfarjani for their significant contributions to the research. Special thanks to Javad Mehri Tekmeh for his advice and valuable insights.

## Conflicts of Interest

The authors declare no conflicts of interest.

## References

[1] Sutiarso S, Coesamin M, Nurhanurawati. [The effect of various media scaffolding on increasing understanding of geometry concepts in elementary school students]. *Journal on Mathematics Education*. 2018; *9*(1): 95-102.

[2] Yaftian N, Gorjipour Shubi, F. [Comparative evaluation of the level of geometric reasoning of ninth grade boys and girls based on the theory of Van Healey]. *Journal of curriculum research.* 2021; *21*(1): 157-177. [In Persian]

[3] Akcay B, Akcay, H. [Effectiveness of Science-Technology-Society (STS) instruction on student understanding of the nature of science and attitudes toward science]. *International Journal of Education in Mathematics, Science, and Technology (IJEMST)*. 2015; *3*(1): 37-45.

[4] Wood D, Bruner JS, Ross G. [The role of tutoring in problem solving]. *Journal of Child Psychology and Psychiatry*. 1976; *17*(2): 89-100.
https://acamh.onlinelibrary.wiley.com/doi/10.1111/j.1469-7610.1976.tb00381.x

[5] Posamentier AS, Krulik S. *Problem solving in mathematics grades 3-6: powerful strategies to deepen understanding.* Corwin Press; 2009. [Accessed 11th Jan 2024].

[6] Pamungkas MD, Rahmawati F, Apriliyani MN. *Teaching spatial geometry with GeoGebra: can it improve the problem-solving skills of prospective mathematics teachers?* Journal of Physics; Conference Series: 2021.
*Journal of physics: conference series, 1918, mathematics and its application:*

[7] Aziz Nejad B, Allah Karami F. [Comparing the effect of ICT-based education with traditional on students' academic enthusiasm]. *Technology of educational journal,* 2019; *13*(1):203-212. [In Persian]
https://doi.org/10.22061/jte.2018.3163.1803

[8] Jacinto H, Carreira S. [Mathematical problem solving with technology: the techno-mathematical Fluency of a student-with-GeoGebra]. *International journal of science and mathematics education,* 2017; 15: 1115–1136.

[9] Mainali B. [Representation in teaching and learning mathematics]. *International Journal of Education in Mathematics, Science, and Technology (IJEMST).* 2021; *9*(1): 1-21. https://doi.org/10.46328/ijemst.1111

[10] Hähkiöniemi M, Francisco J, Lehtinen1 A, Nieminen P, Pehkonen, S. [The interplay between the guidance from the digital learning environment and the teacher in supporting folding back]. *Educational studies in mathematics*, 2023; *112*:461–479. https://doi.org/10.1007/s10649-022-10193-x

[11] Clark Wilson A, Robutti O, Thomas M. [Teaching with digital technology]. ZDM **.** 2020*;* 52: 1223–1242.
https://doi.org/10.1007/s11858-020-01196-0

[12] Hamidi SM. *The effect of software on learning mathematics (geometry)* [Master's thesis]. St Lucia, Qld: *Shahid Chamran university of Ahvaz*; 2011.

[13] Celen Y. [Student opinions on the use of GeoGebra software in mathematics teaching]. *TOJET: The Turkish online Journal of educational technology*. 2020; *19*(4): 84-88.

[14] Ng O-L, Shi L,Ting F. [Exploring differences in primary students' geometry learning outcomes in two technology-enhanced environments: dynamic geometry and 3D printing]. *International Journal of STEM Education*. 2020.
https://doi.org/10.1186/s40594-020-00244-1

[15] Stols G, Kriek J. [Why do not all maths teachers use dynamic geometry software in their classrooms?]. *Australian journal of educational technology*. 2011; *27*(1): 137-151.

[16] Polya G, Szego G. Problems and Theorems in Analysis. Springer-verlag Berlin: heidelberg GmbH; 1972.

[17] Saif AA. Modern educational psychology. *Doran publisher*; 2011: 219. [In Persian]

[18] Ghaleeh Novi F. [Constructivist approach in schools]. *Transformation and education quarterly.* 2018; *4*: 9-11. [In Persian]

[19] Woolfolk A. *Educational psychology (12th edition).* Ohio: The Ohio state university;2012.

[20] Azimpour S. [The role of drawing in geometric reasoning]. *Survey in basic science education.* 2019; *4*(13): 35-43. [In Persian]

[21] Solvang L, Haglund J. [Learning with friction-students' gestures and enactment in relation to a GeoGebra simulation]. *Research in science education.* 2022; *52*: 1659–1675. https://doi.org/10.1007/s11165-021-10017-7

[22] Zanganeh H, Saedy N. [The effect of three-dimensional educational simulation of spatial concepts of geometry lesson on learning-memorization of third year high school students]. *Education strategies in medical sciences.* 2016; 9(6): 431-438. [In Persian]

[23] Mohammadi Z. [Examples in mathematics education]. *Journal of the development of mathematics education.* 2019; *133*: 13-16. [In Persian]

[24] Atkinson RK, Derry SJ, Renkl A, Wortham D. [Learning from examples: instructional principles from the worked examples research]. *Review of educational research***.** 2000; *70*(2): 181-214.
https://journals.sagepub.com/doi/10.3102/00346543070002181

[25] Ozgur Z, Ellis AB, Vinsonhaler R, Dogan MF, Knuth E. [From examples to proof: Purposes, strategies, and affordances of example use]. *Journal of Mathematical Behavior.* 2019; *53*: 284–303.

[26] Rowland T. [The purpose, design and use of examples in the teaching of elementary mathematics]. *Educational Studies in Mathematics*. 2008; *69*(2): 149-163. https://doi.org/10.1007/s10649-008-9148-y

[27] Reyhani E, Mesgarani H, Farmehr F. [Teaching the role of dynamic geometry software in solving the problem of geometry with a focus on geometry]. *Journal of Education Technology.* 2009; *3*(2):73-86. [In Persian]

[28]Gholam Azad S.[The trace of realistic mathematics education in the school mathematics in Iran]. *Journal of Theory and Practice in Curriculum.* 2014; 2 (3): 47-70. [In Persian]

[29] Azimpour S, Vahedi Sh, Faghfouri M. [A comparison of the effects of teaching with dynamic and static graphic images methods on geometric attitude and anxiety of students]. *Biquarterly Journal of Cognitive Strategies in Learning.* 2021; *9*(16): 57-72. [In Persian]

[30] Jafari L, Mahdavi Nasab Y. [Investigating the use of GeoGebra software in teaching mathematics to students]. *The first national conference of Farda school:2018*. [In Persian]

[31] Behzadi Azad P, Mollazehi A. [Interactive teaching of mathematics with GeoGebra from Tehran to Sib and Sooran]. *Development of educational technology*. 2020; 36(5): 45-48. [In Persian]

[32] Olsson J. [Relations between task design and students' utilization of GeoGebra]. *Digital experiences in mathematics education*. 2019; *5*: 223–251. https://doi.org/10.1007/s40751-019-00051-6

[33] Mehri-Tekmeh M, Fariborzi- Araghi MA, Reyhani E. [The effectiveness of teaching, via constructed examples by using GeoGebra software in learning high school geometry theorems]. *Technology of education journal*. 2023; 17(1): 23-38. **[In** Persian] https://doi.org/10.22061/tej.2022.8711.2718

[34] Turgut M. [Reinventing geometric linear transformations in a dynamic geometry environment: multimodal analysis of student reasoning]. *International journal of science and mathematics education.* 2022; *20*: 1203–1223. https://doi.org/10.1007/s10763-021-10185-y

[35] van den Heuvel-Panhuizen M, van Zanten M.[ Realistic mathematics education: A brief history of a longstanding reform movement]. *Mediterranean Journal for Research in Mathematics Education.* 2020;17: 65-73.

[36] Oldknow A, Taylor R. *Teaching mathematics with ICT, Continuum*. London;2000.

[37] Rauf Moghadam L.. *New strategies in psychology education from childhood to adolescence*. Iran Tehran: Publishing Farhang Sabz;2010. [In Persian]

[38] Yao X, Zhao J. [Chinese mathematics teachers use of digital technologies for instruction: A survey study]. *Eurasia journal of mathematics, science and technology education*. 2022; 18(8): Article No: em2135. https://doi.org/10.29333/ejmste/12209

[39] Aminifar E, Saleh Sedghpour B, Zadehdabagh H. [The effect of computer games on students' motivation and mathematical progress]. *Scientific research journal of education technology*. 2012; *6*(2): 81-88. [In Persian] https://doi.org/10.22061/tej.2012.205

[40] Rajabian-Dehzire M, Dortaj F, Maghami HR, Namvar K. [The effect of educational simulations relating to computer on academic goal orientation and students' learning performance]. *Journal of school psychology.* 2021; *10*(1): 104-130. [In Persian]

[41] Yaftian N, Safabakhsh A. [Teaching and learning geometry: Introduction to Van Healey's theory]. *Mathematics and society quarterly.* 2019; *4*(3): 9-22. [In Persian]

[42] Fadaee B, Ghahramani H. [Investigating the impact of several factors (Interest, educational tools, teaching methods) on learning geometry]. *Research in mathematics education.* 2021; *2*(2): 47-55. [In Persian]

[43] Guven B, Kosa T. [The effect of dynamic geometry software on student mathematics teachers' spatial visualization skills]. *Electronic version*. *The Turkish online journal of educational technology to get.* 2008; *7*(4):100-107.

[44] Hodovaniuk T, Makhometa T, Tiahai I, Voznosymenko D, Dubovyk V. Use of the dynamic mathematical program of GeoGebra in classes in mathematical disciplines in the conditions of blended learning. 2022.

[45] Hohenwarter M, Jarvis D, Lavicza Z.. [Linking geometry, algebra, and mathematics teachers: GeoGebra software and the establishment of the international GeoGebra institute]. *International journal for technology in mathematics education*. 2009;16(2): 83-87. *International journal for technology in mathematics education*, 16, 2, 83-87.

[46] Ridha MR, Pramiarsih EE, Widjajani. *The use of GeoGebra software in learning geometry transformation to improve students' mathematical understanding ability*. Journal of physics: conference series: *2020.* 1477.

[47] Pamungkas MD, Nugroho H. [Implementation of space geometry learning using GeoGebra to improve problem solving skills]. *Mapan journal matematika & pembelajaran.* 2020; *8*(2):224-235.

[48] Sugandi AI, Bernard M. *Application of GeoGebra software to improve problem-solving skills in analytic geometry in prospective teacher students*. Journal of physics: Conference series: 2020. 1657. [In Persian]

[49] Rezaee M, Mohammadzadeh E. [Investigating the role of educational examples in teaching and learning mathematics]. *The third national conference of modern studies and research in the field of educational sciences and psychology*. 2017. [In Persian]

[50] Lynch AG, Lockwood E, Ellis AB. [Comprehensive example generation: mathematicians' uses of examples when developing conjectures]. *Research in Mathematics Education: 2022*. https://doi.org/10.1080/14794802.2022.2156586

[51] Reyhani E, Bakhshalizadeh Sh, Vasheghani-Farahani A. [A study of high school math teachers' knowledge of math examples and how to use it to introduce a concept]. *Quarterly journal of educational innovations*. 2013; *46*:125-155. [In Persian]

[52] Trafton JG, Reiser BJ. [The contributions of studying examples and solving problems to skill acquisition]. *Academia.edu*;1994.

[53] Eviliasani K, Sabandar J, Fitriani N. [Problem-based learning assisted by GeoGebra to improve students' mathematical understanding]. *Al-Ishlah: journal Pendidikan.* 2022; *4*(1): 85-98.

[54] Aminifar E, Saleh Sedghpour B, Valinejad F. [The Role of Technology on the Mathematics Learning]. *Scientific research journal of education technology.* 2011; *5*(3): 159-166. [In Persian] https://doi.org/10.22061/tej.2011.234

[55] Mukamba E, Makamure C. [Integration of GeoGebra in teaching and learning geometric transformations at ordinary level in Zimbabwe]. *Contemporary mathematics and science education*. 2020; *1*(1): ep20001.

[56]Samieerad MS, Shahraky A. [Artificial intelligence in education with an emphasis on mathematics]. *The first international conference on management, education and training researches in education.* 2023. [In Persian]

[57] Arbian N, Shukor N A. [The effects of GeoGebra on student achievement]. *Procedia - Social and Behavioral Sciences*. 2015; *172*:208 – 214.

[58] Shadaan P, Leong KE. [Effectiveness of using GeoGebra on students' understanding of learning circles]. *Malaysian Online Journal of Educational Technology.* 2013; 1(4): 1-11.

[59] Zengin Y, Tatar E. [Integrating dynamic mathematics software into cooperative learning environments in mathematics]. *Educational Technology & Society*. 2017; 20(2): 74-88.

[60] Joshi DR, Singh KB. [Effect of using GeoGebra on eighth-grade students' understanding in learning linear equations]. *Editorial from Bronisław Czarnocha*. 2020: 12(3): 76-83.

[61] Martinovic D, Manizade AG. [Teachers Using GeoGebra to Visualize and Verify Conjectures About Trapezoids]. *Canadian Journal of Science, Mathematics and Technology Education.* 2020; 20: 485–503. https://doi.org/10.1007/s42330-020-00103-9

## AUTHOR(S) BIOSKETCHES

**Monireh Mehri Tekmeh** holds a Bachelor's degree in Mathematics from the University of Sistan and Baluchistan and a Master's degree in Pure Mathematics from Vali-e-Asr University of Rafsanjan. She is currently pursuing a Ph.D. in Mathematics Education at Islamic Azad University, Central Tehran Branch. With 28 years of experience in the field, Monireh serves as the Mathematics Secretary for the third district of Isfahan. She is also a teacher and guest lecturer at Farhangian University.

**Mehri-Tekmeh M., PhD student in Mathematics Education, Islamic Azad University, Central Tehran Branch, Tehran, Iran**
*th.of.mmt@gmail.com*

**Mohammad Ali Fariborzi Araghi** is a professor and member of the Faculty of Mathematics and Computer Science Department of Islamic Azad University, Central Tehran Branch. He completed his undergraduate studies in the field of mathematics at Khwarazmi University of Tehran (formerly Teacher Training). He received his master's degree from the Islamic Azad University of Karaj branch and his Ph.D. Degree from Islamic Azad University, Science and Research branch, applied mathematics field, numerical analysis major.

**Fariborzi Araghi, M.A. Full Professor and Faculty Member, Department of Mathematics and Computer Science, Islamic Azad University, Central Tehran Branch, Tehran, Iran**
*m_fariborzi@iauctb.ac.ir ;*
*fariborzi.araghi@gmail.com*

**Ebrahim Reyhani** is an associate professor at the Department of Mathematics, Faculty of Sciences, Shahid Rajaei Teacher Training University. He received his bachelor's degree in mathematics from Khwarazmi University in 1368 and his master's degree in pure mathematics from Shahid Beheshti University in 1373. He received his doctorate from Moscow State University in the field of mathematics education.

**Reyhani E. Associate Professor, Department of Mathematics, Faculty of Sciences, Shahid Rajaee Teacher Training University, Tehran, Iran**
*e_reyhani@yahoo.com*

**Citation (Vancouver):** Mehri-Tekmeh M, Fariborzi-Araghi M.A, Reyhani E. [Enhancing teaching of longitudinal relations theorems in Iranian high school geometry: Integrating GeoGebra software and practical examples]. *Tech. Edu. J. 2025; 19(1): 195-214*

https://doi.org/10.22061/tej.2025.10662.3040